A Multi-set Approach for Direction Finding Based on Spatially Displaced Electromagnetic Vector-Sensors

Meijiao Ji, Xiaofeng Gong, and Qiuhua Lin
School of Information and Communication Engineering
Dalian University of Technology, Dalian 116023, China
E-mail: xfgong@dlut.edu.cn

Introduction

- Configuration of collocated electro-magnetic vector-sensors (EMVS)
  - Mutual coupling exists
  - Complicated hardware implementation
  - Allow the use of the vector cross-product direction finding scheme

- Configuration of spatially displaced electro-magnetic vector-sensors
  - Mutual coupling is reduced
  - Hardware cost is lower
  - Still allow the use of the vector cross-product direction finding scheme

- Spatially displaced electro-magnetic vector-sensors (SD-EMVS) array
  - \( \alpha \) is the counter-clockwise rotation angle
  - SD-EMVS is arranged in uniform line array.

- **scenario 1**
The latent source variables \( s^{(1)}_r \) and \( s^{(2)}_r \) for the \( r \)-th and \( r \)-th SD-EMVS’s are identical:
\[
\begin{align*}
    s^{(1)}_r &= s^{(2)}_r \\
    r &= 1, 2, ..., R
\end{align*}
\]

- **scenario 2**
The latent source variables \( s^{(1)}_r \) and \( s^{(2)}_r \) for the \( r \)-th and \( r \)-th SD-EMVS’s are not identical but correlated:
\[
\begin{align*}
    s^{(1)}_r &= s^{(2)}_r + \epsilon \\
    0 < \rho^{(1,2)} < 1
\end{align*}
\]

- Data model for SD-EMVS array within a single-set framework
\[
x(t) = \sum_{r=1}^{R} a_r s_r(t) + n(t) = A x(t) + n(t)
\]
- Modify the outputs to a large array for scenario 1
- Process each dataset separately for scenario 2

- Data model for SD-EMVS array within a multi-set framework
\[
x(t) = \sum_{r=1}^{R} a^{(r)}(t) + n(t)
\]
- Multiple datasets are jointly analyzed as a group for both scenario 1 and 2

Proposed algorithm

We reconsiders the SD-EMVS array signals within the multi-set framework where each SD-EMVS output is re-interpreted as one single dataset, and proposes to use the newly developed multi-set algorithm, namely generalized non-orthogonal joint diagonalization (GNJD) to estimate the steering vectors. Vector cross-product and optimization are used to extract the DOA’s.

- **Input:** observation \( x(t) \in \mathbb{C}^M, r = 1, 2, ..., R \) , snapshot \( T \)
- **Implementation:**
  - Calculate the cross covariance matrices \( C_{x_n,t} = E(x^{(n)}(k) [x^{(n)}(k)]^* ) \)
  - Apply generalized non-orthogonal joint diagonalization (GNJD) to identify steering vector: \( A^{(n)} \)
  - Convert \( A^{(n)} \in \mathbb{C}^{L \times M} \) to \( \tilde{A}^{(n)} \in \mathbb{C}^{L \times L} \) according to projection matrix
  - Extract the DOA parameters by cross-product and optimization
- **Output:** estimated DOA parameters: \( \theta \) and \( \phi \)

Simulation

- Observed signal data with Gaussian noise:
\[
X_r = \sigma_r A x_r(t) + \sigma_s n_r(t), r = 1, 2, ..., R
\]
- \( \sigma_r \) and \( \sigma_s \) denote the signal and noise power, respectively
- \( \text{SNR} = -10\log_{10}(\sigma_r/\sigma_s) \)

- DOA parameters:
  - \( \theta \) \( \gamma \)
  - \( \eta \)
  - Overall Root Mean Squared Angular Error (RMSAE): \( \chi = M \sum_{r=1}^{R} \sqrt{E[\arccos(\hat{a}^* \hat{a})]} \)

- **Simulation 1:** the source signals received by different SD-EMVS are identical: \( \alpha^{(r, s)} = \alpha^{(r, t)} \) with \( 1 \leq r, s \leq R \)
  - inter-spacing between EMVS’s is \( 1.5\lambda \)
  - spread distance of dipoles/loops is \( 0.7\lambda \)
  - \( M = 3 \) \( R = 4 \)

- **Simulation 2:** the \( m \)-th incident source signal received by \( r \)-th and \( r \)-th SD-EMVS are not identical but correlated: \( \alpha^{(m, r)} \)
  - inter-spacing between EMVS’s is \( 1.0\lambda \)
  - spread distance of dipoles/loops is \( 0.7\lambda \)
  - \( M = 3 \) \( R = 4 \)

Conclusions

- Numerical simulation results demonstrate that multi-set scheme could offer better performance compared with the three other schemes based on single-set, particularly at low SNR.
- In practical scenario that the incident sources received by different EMVS are not identical but correlated, multi-set approach provides more accurate estimates than single-set methods significantly.